Math 515-1: Derived commutative rings
Problem set 04

1. Let $R$ be a commutative ring and let $\mathrm{GrMod}_R^\heartsuit = \mathrm{Fun}(\mathbf{Z}^\delta, \mathrm{Mod}_R^\heartsuit)$ be the abelian category of graded static $R$-modules. There is a monoidal structure on $\mathrm{GrMod}_R^\heartsuit$ arising from Day convolution with respect to $+$ on $\mathbf{Z}^\delta$ and the usual tensor product on $\mathrm{Mod}_R^\heartsuit$. First, classify the symmetric monoidal structures on $\mathrm{GrMod}_R^\heartsuit$ extending this monoidal structure. Second, classify the symmetric monoidal structures on $\mathrm{Ch}^\bullet(R)$, the abelian category of of cochain complexes of static $R$-modules, such that the forgetful functor $\mathrm{Ch}^\bullet(R) \to \mathrm{GrMod}_R^\heartsuit$ admits a symmetric monoidal structure. You can and use the "classical" definition of symmetric monoidal 1-categories; this is not a problem about $\infty$-categories. (Suggested by a conservation with Dima Tamarkin.)

2*. Find an $\mathbf{E}_\infty$-algebra $R$ over $\mathbf{F}_2$ such that $\pi_* R \cong \mathbf{F}_2[u]$ where $|u| = 2$ and where $Q^{2s}(u) = u^{s+1}$ for all odd $s > 0$ and $Q^{2s}(u) = 0$ otherwise. (As far as I am aware, this is an open problem. See [1, Question 5.18] for details.)

3*. Is every $\mathbf{Z}$-linear cdga $R^\bullet$ quasi-isomorphic (as a cdga) to one such that $R^n$ is projective for each $n \in \mathbf{Z}$? (Suggested by a conservation with Vladimir Shein.)

## References

[1] Jun Hou Fung, *Strict units of commutative ring spectra*, arXiv preprint arXiv:1911.11850 (2019).

Department of Mathematics, Northwestern University
antieau@northwestern.edu