Math 515-1: Derived commutative rings
Problem set 03

1. For any $j, n, s \geqslant 0$, compute $\mathrm{Sq}^{2s}(c_j) = \mathrm{Q}^{-2s}(c_j)$ in $\mathrm{H}^*(\mathrm{BU}_n; \mathbf{F}_2) \cong \mathbf{F}_2[c_1, \ldots, c_n]$ (where $|c_j| = 2j$).

2. For any $j, n, s \geqslant 0$, compute $\mathrm{Sq}^{s}(w_j) = \mathrm{Q}^{-s}(w_j)$ in $\mathrm{H}^*(\mathrm{BO}_n; \mathbf{F}_2) \cong \mathbf{F}_2[w_1, \ldots, w_n]$ (where $|w_j| = j$).

3. Compute the $\mathbf{F}_2$-cohomology of the symmetric group Σ_3 as a graded-commutative ring. Compute the action of the Steenrod operations on the generators.

4. Compute the $\mathbf{F}_2$-cohomology of the symmetric group Σ_4 as a graded-commutative ring. Compute the action of the Steenrod operations on the generators.

5. Compute the $\mathbf{F}_2$-cohomology of the Eilenberg–Mac Lane anima $K(\mathbf{Z}, 3)$ as a graded-commutative ring. Compute the action of the Steenrod operations on the generators.

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